Q-FUZZIFICATION OF FERMATEAN FUZZY POINTS AND LOCAL PROPERTIES OF UNCERTAINTY SUBGROUP

[1]Rathinam Nagarajan, [2]Samidurai Vaiyapuri Manemaranm,[3]Kalairajan Balamurugan. [4]Chandrasekaran Dillirani

1.Department of Mathematics, J.J College of Engineering and Technology, Sowdambikka group of institutions, Tiruchirappalli- 620009, Tamilnadu, India. E-mail: rajenagarajan1970@gmail.com
2.Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai, Tamilnadu, India.E.mail: svmanemaran@gmail.com .
3.Department of Mathematics, M.A.M School of Engineering (Autonomous)m Tiruchirappalli-621105, Tamilnadu, India.E-mail: bala.algebra@gmail.com
4.Department of Mathematics, J.J College of Engineering and Technology, Sowdambikka group of institutions, Tiruchirappalli- 620009, Tamilnadu, India.E-mail: kanibala21@gmail.com

Abstract: The concept of a fermatean set as first proposed by senapati and yager as the extension of intuitionistic fuzzy set and Pythagorean fuzzy set. In this paper, we studied the notion of a fermatean Q-fuzzy point to study for the first time the notion of fermatean Q-fuzzy subgroups and its local properties. This new concept showed us to reformulate all the mathematical properties of fermatean Q-fuzzy subgroups structure.

Key words: Q-fuzzy set, fuzzy point, intuitionstic fuzzy set, Pythagorean fuzzy set, Fermatean Q-fuzzy set. fermatean Q-fuzzy subgroup.

AMS classification (2020): 03F55, 06D72, 08A72.

1.Introduction: The concept of fuzzy set is first introduced by L.A.Zadeh in 1965 [24]. After the tremendous development intuitionistic fuzzy set [4], pythagoren fuzzy set [23], (3,2) fuzzy set[15], picture fuzzy set [14], Neutrosophoic fuzzy set [19] was studied. Recently fermatean fuzzy set [19] was studied by Yager and its application is briefly explained by [23]. Due to the inadequacy of FST in the sense that it admits only the membership grade, intuitionistic fuzzy sets (IFSs) was introduced [5, 6] by incorporating membership and non-membership grades with the chance for hesitation margin. Various properties of IFSs were presented [4,5] and IFSs have been applied in real-life problems [4, 5, 6]. Consequently, Biswas [8, 9] introduced intuitionistic fuzzy subgroups (IFSGs) based on IFSs, Ahn et al. [2] deliberated on sublattice of lattice of IFSGs of a group, some properties of IFSGs were discoursed in [1]. In addition, Yuan et al. [22] shared some light on the description of IFSGs, Bal et al. [7] presented a brief note of kernel subgroups on IFGs and derived some properties of IFGs. Senapati.T [19] introduced fermatean fuzzy sets in 2020, an extension of intuitionistic fuzzy set (IFSs) and pythagorean fuzzy set (PFSs). AVs and NAVs of fermatean fuzzy sets reveal their dependence on greater powers with sum of cubes is less than 1. Ibrahim.H.Z [15] introduced and applied n, m-rung orthopair fuzzy sets in MCDM.. The concepts of fuzzy modules and fuzzy sub-modules was introduced by Negoita and Ralescu [16] in 1975. The concept of essential fuzzy modules was introduced by Hadi [13] in 2000. Using this idea. Abbas established the concept of essential fuzzy sub-modules and uniform fuzzy modules in 2012 Nagarajan and Solairaju and [17], new Structure and Constructions of Q-fuzzy groups In this paper, we studied the notion of a fermatean Q-fuzzy point to study for the first time the notion of fermatean Q-fuzzy subgroups and its local properties. This new concept showed us to reformulate all the mathematical properties of fermatean Q-fuzzy subgroups structure.

2. Preliminaries

In this section, we will present some notions and results to use them in the sequal.

Definition-2.1: [24] Let X be an universe of discourse. Then a fuzzy set A is an object having the following formulation: $A = \{(x, \mu_A(x))/x \in X\}$ where $\mu_A: X \to [0,1]$ and $\mu_A(x)$ is called the member degree of x in X.

Example 2.2: Let $X = \{a, b, c\}$. Then the fuzzy set is defined as

x	a	b	c
$\mu_A(x)$	0.3	0.2	0.7

Definition-2.3: [17] A Q-fuzzy set is defined as a mapping $\mu: X \times Q \to [0,1]$, assigning a degree of membership between 0 and 1 to each element $x \in X$ for each parameter $q \in Q$.

Definition-2.4:[17] Let X be a non-empty set. An intuitionistic Q-fuzzy set (IQFS for short) of X defined as an object having the form $A = \{(x, J(x,q), K(x,q))/x \in X, q \in Q\}$, where $J: X \times Q \to [0,1]$ and $K: X \times Q \to [0,1]$ denote the degree of membership (namely $J(x,q)$) and the degree of non-membership (namely $K(x,q)$) of each element $x \in X$ it to the set A, respectively and $0 \leq J(x,q) + K(x,q) \leq 1$ for each $x \in X$ and $q \in Q$. For the sake of simplicity we shall use the symbol $A = (J, K)$ for the intuitionistic Q-fuzzy set
$A = \{(x, J(x,q), K(x,q))/x \in X, q \in Q\}$.

Definition-2.5: [23]Let X be a non-empty set. A pythagorean fuzzy set (PFS) A of an universal set X is of the form $A = \{\langle x, J(x), K(x)\rangle/x \in X\}$ where $J: X \to [0,1]$ and $K: X \to [0,1]$ and
$0 \leq J^2(x) + K^2(x) \leq 1$ for each $x \in X$.

Definition-2.6: [23] Let X be a non-empty set. Let (J_1, K_1) and (J_2, K_2) be two pythagorean fuzzy subsets of , then

(i) The intersection set $(J, K) = (J_1, K_1) \cap (J_2, K_2)$ is defined by
$J = J_1 \wedge J_2 = min\{J_1, J_2\}$ and $K = K_1 \vee K_2 = max\{K_1, K_2\}$.

(ii) The intersection set $(J, K) = (J_1, K_1) \cup (J_2, K_2)$ is defined by

$$J = J_1 \vee J_2 = max\{J_1, J_2\} \text{ and } K = K_1 \wedge K_2 = min\{K_1, K_2\}.$$

Definition-2.7: [19] Let X be a non-empty set. A fermatean fuzzy set (FFS) A of an universal set X is of the form $A = \{(x, J(x), K(x))/x \in X\}$ where $J: X \to [0,1]$ and $K: X \to [0,1]$ and $0 \leq J^3(x) + K^3(x) \leq 1$ for each $x \in X$.

3. Main Results

Definition-3.1: Let $(G, +)$ be a group and (J, K) be a fermatean fuzzy set(FFS). Then (J, K) is said to be a fermatean Q-fuzzy subgroup(FQFSG) of G if for all $x, y \in G$, the following conditions hold;

(FQFSG1): $J^3(x+y)_q \geq min\{J^3(x,q), J^3(y,q)\}$

(FQFSG2): $K^3(x+y)_q \leq max\{K^3(x,q), K^3(y,q)\}$

(FQFSG3): $J^3(-x)_q \geq J^3(x,q)$

(FQFSG4): $K^3(-x)_q \leq K^3(x,q)$, where $q \in Q$.

Example-3.2: Let $X = \{x, 0.3, 0.4\}$.

Here $J^3(x) + K^3(x) = (0.3)^3 + (0.4)^3 = 0.027 + 0.064 = 0.091 < 1$.

And $\pi_A(x) = 1 - 0.091 < 1$ is called the degree of refusal membership of x in X.

Preposition-3.3: Let $I = (J, K)$be a FQFSG of a group $(G, +)$. Then the following holds:

(i) $J^3(e)_q \geq J^3(x,q)$ and $K^3(e)_q \leq K^3(x,q)$ for all $x \in X, q \in Q$.

(ii) $J^3(e)_q = J^3(x,q)$ and $K^3(e)_q = K^3(x,q)$ for all $x \in X, q \in Q$,

where 'e' is the identity element in G.

Preposition-3.4: Let $I = (J, K)$be a FFS of a group $(G, +)$. Then $I = (J, K)$ is a FQFSG of a group $(G, +)$ if and only if

$$J^3(x-y)_q \geq min\{J^3(x,q), J^3(y,q)\} \text{ and}$$

$$K^3(x-y)_q \leq max\{K^3(x,q), K^3(y,q)\}, \forall x, y \in G.$$

Definition-3.5: Let 'E' be a set. Let $x \in X$, $r \in [0,1]$ and $(\alpha, \beta) \in [0,1]$ such that $\alpha^3 + \beta^3 = r^3 \leq 1$. Then the fermatean fuzzy subset $x_{(\alpha,\beta)} = (t_\alpha, 1 - t_{1-\beta})$ is called a fermatean Q-fuzzy point in X with;

$$t_\alpha(y)_q = \begin{cases} \alpha, \text{if } x = y \\ 0, \text{if } x \neq y \end{cases} \text{ and } (1 - t_{1-\beta})(y)_q = \begin{cases} \beta, \text{if } x = y \\ 0, \text{if } x \neq y \end{cases}$$

with the function $r_{x(\alpha,\beta)}: X \times Q \to [0,1]$ given by

$$r_{x(\alpha,\beta)}(y)_q = \begin{cases} r, \text{if } x = y \\ 1, \text{if } x \neq y \end{cases}$$

For reasons of simplicity, we can define a fermatean fuzzy point in the following style.

Definition-3.6: Let $\alpha, \beta \in [0,1]$ with $\alpha^3 + \beta^3 \leq 1$. A fermatean Q-fuzzy point written as $x_{(\alpha,\beta)}$ is defined to be an fermatean Q-fuzzy subset of X, given by

$$x_{(\alpha,\beta)}(y)_q = \begin{cases} (\alpha, \beta), \text{if } x = y \\ (0,1), \text{if } x \neq y \end{cases}$$

A fermatean Q-fuzzy point $x_{(\alpha,\beta)}$ is said to belong to FFS(J, K) denoted by $x_{(\alpha,\beta)} \in (J, K)$ if $J^3(x)_q \geq \alpha$ and $K^3(x)_q \leq \beta$.

Preposition-3.7: Let $(G, +)$ be a group and $H \subset G$, we note by χ_H the characteristic function of the set H. Then, H is te subgroup of G if and only if $(\chi_H, 1 - \chi_H)$ is a non-zero fermatean Q-fuzzy subgroup of G.

Proof: Suppose that H is a subgroup of G. Let 'e' be the neutral element of G, then $e \in H$ and $\chi_H(e) = 1$.

So $(\chi_H, 1 - \chi_H) \neq \emptyset$.

Let $x, y \in H$ and $q \in Q$. If $x \in H$ and $y \in H$, as H is a subgroup of G, then $-y \in H$ and $x + y \in H$ as result $x - y \in H$.

Therefore, ${\chi_H}^3(x - y)_q = 1 = min\{{\chi_H}^3(x, q), {\chi_H}^3(y, q)\}$.

If $x \notin H$ or $y \notin H$, we have ${\chi_H}^3(x, q) = 0$ or ${\chi_H}^3(y, q) = 0$.

Hence, $min\{{\chi_H}^3(x, q), {\chi_H}^3(y, q)\} = 0 \leq {\chi_H}^3(x - y)_q$.

In this same way we show that

$$(1 - \chi_H)^3(x - y)_q \leq min\{(1 - \chi_H)^3(x, q), (1 - \chi_H)^3(y, q)\}$$

Consequently, $(\chi_H, 1 - \chi_H)$ is a fermatean Q-fuzzy subgroup of G.

There exists $x \in G$ such that $\chi_H(x, q) = 1$, which proves that $H \neq \emptyset$.

Moreover, if $x, y \in H$, ${\chi_H}^3(x, q) = {\chi_H}^3(y, q) = 1$.

Hence, as ${\chi_H}^3(x - y)_q \geq min\{{\chi_H}^3(x, q), {\chi_H}^3(y, q)\}, {\chi_H}^3(x - y)_q = 1$ and as a result, $x - y \in H$.

Therefore H is a subgroup of G.

Using the definition-3.1, we will have the following theorem.

Theorem-3.8: Let $(G, +)$ be a group and 'A' be s fermatean Q-fuzzy subset of G. Then A is a FQFSG of G. Then A is a FQFSG of G if and only if

1. $x_{(\alpha_1,\beta_1)} \in A$, $y_{(\alpha_2,\beta_2)} \in A$ implies $(x + y)_{(\alpha_1 \wedge \alpha_2, \beta_1 \wedge \beta_2)} \in A$.
2. $x_{(\alpha,\beta)} \in A$ implies $(-x)_{(\alpha,\beta)} \in A$

Proof: Sufficient Part:

Let's show that the conditions of the definition of the FQFSG are verified.

Let $\alpha = min\{{J_A}^3(x, q), {J_A}^3(y, q)\}$,

$\beta = max\{{K_A}^3(x,q),{K_A}^3(y,q)\}$, then $\alpha^3 + \beta^3 \leq 1$ and

${J_A}^3(x,q) \geq \alpha,\ {J_A}^3(y,q) \geq \alpha$ and

${K_A}^3(x,q) \leq \beta, {K_A}^3(y,q) \leq \beta$.

Then $(x+y)_{(\alpha,\beta)} \in A$.

There fore ${J_A}^3(x+y)_q \geq \alpha^3 = min\{{J_A}^3(x,q),{J_A}^3(y,q)\}$ and

$${K_A}^3(x+y)_q \leq \beta^3 = max\{{K_A}^3(x,q),{K_A}^3(y,q)\}.$$

As a result, A is a FQFSG of G.

Necessary Part:

Suppose that A is a FQFSG of G.

Let $x_{(\alpha_1,\beta_1)} \in A$, $y_{(\alpha_2,\beta_2)} \in A$.

So, ${J_A}^3(x,q) \geq \alpha_1, {J_A}^3(y,q) \geq \alpha_2$ and

${K_A}^3(x,q) \leq \beta_1, {K_A}^3(y,q) \leq \beta_2$.

Hence,

$$\begin{aligned}{J_A}^3(x+y,q) &\geq min\{{J_A}^3(x,q),{J_A}^3(y,q)\}\\ &\geq min\{\alpha_1,\alpha_2\} \text{ and}\\ {K_A}^3(x+y)_q &\leq max\{{K_A}^3(x,q),{K_A}^3(y,q)\}\\ &\leq max\{\beta_1,\beta_2\}.\end{aligned}$$

Then $(x+y)_{(\alpha_1\wedge\alpha_2,\beta_1\vee\beta_2)} \in A$.

Moreover, ${J_A}^3(-x,q) \geq {J_A}^3(x,q)$ and ${K_A}^3(-x,q) \leq {K_A}^3(x,q)$.

And because $x_{(\alpha,\beta)} \in A$.

So, ${J_A}^3(-x,q) \geq {J_A}^3(x,q) \geq \alpha$ and

${K_A}^3(-x,q) \leq {K_A}^3(x,q) \leq \beta$, as a result $(-x)_{(\alpha,\beta)} \in A$.

Now, we are going to propose some local properties on the FQFSG using the above theorem.

Theorem-3.9: Let $(G,+)$ be a Q-fuzzy subgroup, and let $A(J,K)$ be a fermatean Q-fuzzy subgroup of G. Then the following properties:

1. Let $x_{(\alpha_1,\beta_1)} \in A(J,K)$, $y_{(\alpha_2,\beta_2)} \in A(J,K)$ be two fermatean Q-fuzzy points, then $(x-y)_{(\alpha_1\wedge\alpha_2,\beta_1\vee\beta_2)} \in A(J,K)$.
2. Let 'e' be the neutral element of G, then $\forall x_{(\alpha,\beta)} \in A(J,K)$, ${J_A}^3(e) \geq \alpha$ and ${K_A}^3(e) \leq \beta$.

Proof: 1. Let $x_{(\alpha_1,\beta_1)} \in A(J,K)$, $y_{(\alpha_2,\beta_2)} \in A(J,K)$ be two fermatean Q-fuzzy points. Then, according to the above theorem, we have $y_{(\alpha_1,\beta_1)} \in A(J,K)$ and

$(x+y)_{(\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2)} \in A(J,K)$.

We have $x - y = x + (-y)$.

So, $(x-y)_{(\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2)} \in A(J,K)$.

2. Let 'e' be the neutral element of G, let $x_{(\alpha,\beta)} \in A(J,K)$ be a fermatean Q-fuzzy point, then ${J_A}^3(x,q) \geq \alpha$ and ${K_A}^3(x,q) \leq \beta$.

Then. According to the proposition-*3.3*, we have

${J_A}^3(e,q) \geq {J_A}^3(x,q)$ and ${K_A}^3(e,q) \leq {K_A}^3(x,q)$.

So, ${J_A}^3(e,q) \geq {J_A}^3(x,q) \geq \alpha$ and ${K_A}^3(e,q) \leq {K_A}^3(x,q) \leq \beta$.

Note-3.10: Note that we have $J_A(e,q) = 1$, $K_A(e,q) = 0$.

Let $P = \{(\alpha,\beta)/(\alpha,\beta) \in [0,1]/\alpha^3 + \beta^3 \leq 1\}$.

Lemma-3.11: Let 'A ' a fermatean Q-fuzzy subset of E, for $(\alpha,\beta) \in P$. Let

$$A_\alpha = \{x \in E/{J_A}^3(x,q) \geq \alpha\} \text{ and}$$
$$A^\beta = \{x \in E/{K_A}^3(x,q) \leq \beta\}$$

then $\big(J_A(x,q), K_A(x,q)\big) = \big(\vee\{\alpha/x \in A_\alpha\},\ \wedge\{\beta/x \in A^\beta\}\big)$ for all $x \in E$.

Proof: Let $A(J,K)$ be a fermatean Q-fuzzy subset of E. We have

$$A_\alpha = \{x \in E/{J_A}^3(x,q) \geq \alpha\} \text{ and}$$
$$A^\beta = \{x \in E/{K_A}^3(x,q) \leq \beta\}.$$

Then, $$\vee\{\alpha/x \in A_\alpha\} = \vee\{\alpha \in [0,1], x \in E, {J_A}^3(x,q) \geq \alpha\} \text{ and}$$
$$\wedge\{\beta/x \in A^\beta\} = \wedge\{\beta \in [0,1], x \in E, {K_A}^3(x,q) \leq \beta\}.$$

Hence, $\big(J_A(x,q), K_A(x,q)\big) = \big(\vee\{\alpha/x \in A_\alpha\},\ \wedge\{\beta/x \in A^\beta\}\big)$ for all $x \in E$.

Theorem-3.12: Let $(G,+)$ be a group and A is a fermatean Q-fuzzy subset of G. Then A is a fermatean Q-fuzzy subgroup of G if and only if A_α and A^β are subgroups of G for every $(\alpha,\beta) \in P$.

Proof: Let A be a Fermatean Q-fuzzy subgroup of G.

We will show that A_α and A^β are subgroups of G.

Let $x, y \in A^\beta$. Then we have ${K_A}^3(x,q) \leq \beta$ and ${K_A}^3(y,q) \leq \beta$.

Therefore, we have

$$\begin{aligned} {K_A}^3(x-y)_q &\leq max\{{K_A}^3(x,q), {K_A}^3(-y,q)\} \\ &\leq max\{{K_A}^3(x,q), {K_A}^3(y,q)\} \\ &\leq \beta \end{aligned}$$

As a result, $x - y \in A^\beta$.

Therefore, A^β is a subgroup of G.

Similarly, we can show that A_α is a subgroup of G.

Conversely, we have

$$\begin{aligned}
{J_A}^3(x+y)_q &= \left\{\vee\{\alpha/x+y\in A_\alpha\}\right\}^3 \\
&\geq \left\{\vee\{\alpha/x\in A_\alpha \text{ and } y\in A_\alpha\}\right\}^3 \\
&= \left\{\vee\left\{\alpha/{J_A}^3(x,q)\geq\alpha, {J_A}^3(y,q)\geq\alpha\right\}\right\}^3 \\
&= \left\{\vee\left\{\alpha/min\left\{{J_A}^3(x,q), {J_A}^3(y,q)\right\}\geq\alpha\right\}\right\}^3 \\
&= min\left\{{J_A}^3(x,q), {J_A}^3(y,q)\right\}
\end{aligned}$$

And

$$\begin{aligned}
{K_A}^3(x+y)_q &= \left\{\wedge\left\{\beta/x+y\in A^\beta\right\}\right\}^3 \\
&\leq \left\{\wedge\left\{\beta/x\in A^\beta \text{ and } y\in A^\beta\right\}\right\}^3 \\
&= \left\{\wedge\left\{\beta/{K_A}^3(x,q)\leq\beta\ , {K_A}^3(y,q)\leq\beta\right\}\right\}^3 \\
&= \left\{\wedge\left\{\beta/max\left\{{K_A}^3(x,q), {K_A}^3(y,q)\right\}\leq\beta\right\}\right\}^3 \\
&= max\left\{{K_A}^3(x,q), {K_A}^3(y,q)\right\}
\end{aligned}$$

And we have

$$\begin{aligned}
{J_A}^3(-x)_q &= \left\{\vee\{\alpha/-x\in A_\alpha\}\right\}^3 \\
&= \left\{\vee\{\alpha/x\in A_\alpha\}\right\}^3 \\
&= {J_A}^3(x,q)
\end{aligned}$$

Also,

$$\begin{aligned}
{K_A}^3(-x)_q &= \left\{\wedge\left\{\beta/-x\in A^\beta\right\}\right\}^3 \\
&= \left\{\wedge\left\{\beta/x\in A^\beta\right\}\right\}^3 \\
&= {K_A}^3(x,q)
\end{aligned}$$

Therefore, A is a Fermatean Q-fuzzy subgroup of G.

Conclusion: In this paper, we have presented a new definition of fermatean Q-fuzzy subgroups(FQFSG) using the concept of fermatean Q-fuzzy points and with the help of these basic concepts. We have exposed to show some local properties on fermatean Q-fuzzy subgroups.

Reference

[1] Ahn, T. C., Hur, K., Jang, K. W., & Roh, S. B. (2006). Intuitionistic fuzzy subgroups. Honam Mathematical Journal, 28(1), 31–44.

[2] Ahn, T. C., Jang, K. W., Roh, S. B., & Hur, K. (2005). A note on intuitionistic fuzzy subgroups. Proceedings of KFIS Autumn Conference 2005, 15(2), 496–499.

[3] Anthony, J. M., & Sherwood, H. (1982). A characterization of fuzzy subgroups. Fuzzy Sets and Systems, 7(3), 297–305.

[4] Atanassov, K. T. (1983). Intuitionistic fuzzy sets. VII ITKR's Session, Sofia, June 1983 (Deposed in Central Science-Technical Library of Bulgaria Academic of Science, 1697/84) (in Bulgarian). Reprinted: International Journal Bioautomation, 2016, 20(S1), S1–S6.

[5] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87–96.

[6] Atanassov, K. T. (1994). New operations defined over the intuitionistic fuzzy sets. Fuzzy Sets and Systems, 61(2), 137–142.

[7] Bal, M., Ahmad, K.D., Hajjari, A.A., & Ali, R. (2022). A short note on the kernel subgroup of intuitionistic fuzzy groups. Journal of Neutrosophic and Fuzzy Systems, 2(1), 14–20.

[8] Biswas, R. (1989). Intuitionistic fuzzy subgroups. Mathematical Forum, 10, 37–46.

[9] Biswas, R. (1997). Intuitionistic fuzzy subgroups. Notes on Intuitionistic Fuzzy Sets, 3(2), 53–60.

[10] B. C. Cuong. Picture fuzzy sets-first results. part 2. Seminar Neuro-Fuzzy Systems with Applications, Preprint 04/2013, Institute of Mathematics, Hanoi, June 2013. [5] B. C. Cuong. Picture fuzzy sets. Journal of Computer Science and Cybernetics, 30(4)(2014), 409–420.

[11] Ejegwa, P. A., Ajogwu, C. F., & Sarkar, A. (2023). A hybridized correlation coefficient technique and its application in classification process under intuitionistic fuzzy setting. Iranian Journal of Fuzzy Systems, 20(4), 103–120.

[12] Fathi, M., & Salleh, A. R. (2009). Intuitionistic fuzzy groups. Asian Journal of Algebra, 2(1), 1–10.

[13] Hadi. I.M, On some special fuzzy ideal of fuzzy ring, Accepted in J. Soc. of Phy-Math(2000).

[14] Hamil. M.A, Semi prime fuzzy modules, Ibn. Al-Haitham J. pure and applied science, 25(1) (2012), 1-10.

[15] H. Z. Ibrahim, T. M. Al-shami, O. G. Elbarbary, (3, 2)-fuzzy sets and their applications to topology and optimal choice, Computational Intelligence and Neuroscience, **2021** (2021), 14 pages.

[16] Nagotia. X.V and Ralescu.D, Application of fuzzy sets and system analysis, Birkhauser, Basel, 1975.

[17]. Nagarajan.R, A.Solairaju New Structure and Constructions of Q-fuzzy groups, Advances in Fuzzy Mathematics, Volume 4, Number 1 (2009), pp. 23–29

[18] Rosenfeld, A. (1971). Fuzzy groups. Journal of Mathematical Analysis and Applications, 35(3), 512–517.

[19] T. Senapti and R.R. Yager, Fermatean fuzzy sets, Journal of Ambient Intelligence and Humanized computing, 11, (2) (2020), 663-674.

[20]. F. Smarandache (2003), Definition of Neutrosophic Logic – A Generalization of the Intuitionistic Fuzzy Logic, Proceedings of the Third Conference of the European Society for Fuzzy Logic and Technology, EUSFLAT 2003, September 10-12, 2003, Zittau, Germany; University of Applied Sciences at Zittau/Goerlitz, 141-146.

[21] Xu, C. (2008). New structures of intuitionistic fuzzy groups. In: Huang, D. S., Wunsch, D. C., Levine D. S., & Jo, K. H. (eds). Advanced Intelligent Computing Theories and Applications. With Aspects of Contemporary Intelligent Computing Techniques. Communications in Computer and Information Science, vol 15. Springer, Berlin, Heidelberg, 145–152.

[22] Yuan, X. H., Li, H. X., Lee, E. S. (2010). On the definition of the intuitionistic fuzzy subgroups. Computers and Mathematics with Applications, 59(9), 3117–3129.

[23] R.R.Yager, Pythagorean fuzzy subsets. In:2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), 2013, 36286152.

[24] Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338–353.