ON WEAKLY REGULAR NEAR-RING AND ITS GENERALIZATION

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Abstract: a near-ring which is both left and right weakly regular is called weakly regular. Kovace [1956] proved that, for commutative weak regularity and regularity are equivalent condition. This paper attempts to generalize the properties of regular near-ring taking the condition of weakly regular near-rings.

Key words: Weak regularity, Principal ideal. Maximal ideal, Centre of Near-ring.

1. Introduction: The regularity concept was researched by Mason [1975], Reddy & Murty [1984] Grenewald and Argac [2005]. Andrunakievich [1990] introduced P- regular near-ring and its properties. Mason [1975] [1998] described the regular and strongly regular for right near ring. Several mathematicians also studies and developed varies concept in this area, namely Dhhena, P. & Elevarasan [2013] developed the regularity concept by introducing near-ring are weakly regular and strongly regular. Some other authors like Wendt, Gerhard [2005], T. Manikantan and S. Ram Kumar [2020] researched and established several results.. Later on, Huseyin Kamaci and Akin Osman Atagun [2022] many works were published on the regularity and strongly regularity and also their relation with prime ideals, nilpotent, idempotent, reduced in near-rings.

1.1. Ideal (Pilz, G. 1983): A non - empty subset I of N is said to be Ideal if:

(i) $(I,+)$ is a Normal subgroup of $(N,+)$.
(ii) $NI \subseteq I$ i. e. ($ai \in I$ for all $i \in I$ and $n \in N$ i. e. I is a left ideal), (Similarly for Right Ideal).
(iii) $(n+i)m - nm \in I$ if $n, m \in N$ and $i \in I$.

Example: The set of all even integer $(Z_e, +, .)$ is an Ideal of the near-ring $(Z, +, .)$.

1.2. Definition (Pilz, G. 1983) Let $(I, +, .)$ and $(J, +, .)$ be Ideal of the Near-ring N. Then $I.J = \{\sum_{i=1}^{n} a_i b_i \mid a_i \in I and\ b_i \in J\}$ is also an Ideal of Near-ring N.

1.3. Principal Ideal (Pilz, G. 1983) An Ideal $(I, +, .)$ of the Near-ring N that generated by a single element $a \in N$ is called Principal Ideal and denoted by (a) or aN and Na which defined by $I = (a) = \{n.a : n \in N\} = Na$.

1.4. Zero Divisor (Pilz, G. 1983): A near-ring N is said to have Zero Divisor, if there exists non - zero elements $a, b \in N$ such that $ab = 0$.

1.5. Maximal Ideal (Pilz, G. 1983): A non-zero Ideal I of near-ring N is called Maximal Ideal. If $I \neq N$ and there exists no proper Ideal of a Near-ring $containing$ I i. e. I is a Maximal ideal of N if $I \neq N$, and if there exists an ideal J in N with $I\,J \subseteq N$, then $J = N$.

1.6. Centre of Near-ring (Pilz, G. 1983): Let N be a right near-ring. Denote by $C(N)$ the multiplicative center of N, and by N_d the set of left-distributive elements of N. In general, $C(N)$ need not be closed under the addition of N. However, the generalized center of N, $C(N) = \{ a \in N: an_d = n_d a \; for \; all \; n_d \in N_d\}$, is always a sub-near-ring of Ncontaining $C(N)$.

1.7. Weakly Regular Near-ring (Argac & Groenewald, 2005): A near-ring is called Left Weakly Regular, if $B^2 = B$ for each left ideal B of N. Similarly, a near-ring is called Right Weakly Regular, if $B^2 = B$ for each right ideal B of N. A near-ring is called Weakly Regular, if $B^2 = B$ for each ideal B of N.

2. Theorem: Every regular near-ring is Weakly regular near-ring.

Proof: Let N be a regular near-ring and I be an Ideal of N. Then $I\,I \subseteq I$. Let $a \in I \subseteq N$. So there exists $b \in N$, such that $a = aba$. Since I is an Idea, Hence $ab \in I$ and $aba \in I\,I$. So $a \in I\,I$ and $I \subseteq I\,I$. Thus , $I\,I = I$ and N is Weakly Regular near-ring.

3. Theorem: The following condition are equivalent from any near-ring N.

(i) N is right weakly regular
(ii) For every $n \in N, n \in (nN)^2$
(iii) For any two right ideal $K_1 \subseteq K_2$ of N, $K_1 K_2 = K_1$.

Proof: (i) $\Longrightarrow$ (ii) Let $n \in N$ and K be thr right ideal generated by n. Then $n \in K = K^2, K = nN$. Therefore $n \in (nN)^2$.

(ii) $\Longrightarrow$ (iii) Taking $n \in K_1$ we have $n \in (nN)^2 \subseteq K_1 K_2$. Hence $K_1 \subseteq K_1 K_2$. Since K_1 is right ideal, so $K_1 K_2 \subseteq K_1$. Therefore $K_1 K_2 = K_1$.

(i)$\Longrightarrow$ (iii) Let I be a right ideal, so $I \subseteq I$ and $I\,I = I$. Then N is a right weakly regular.

4. Theorem: If A is proper two sided ideal of a right weakly regular near-ring N, then each element of A is left zero divisor.

Proof: If the element x of A is not a left zero divisor, then x is invertible and equation $xN = xNxN$ yields. $N = NxN \subseteq A$ which is contradiction. Then each element of A is left zero divisor.

5. Theorem: The centre of any right weakly regular near-ring is regular.

Proof: If $x \in C,$ is a centre of right weakly regular near-ring N, then $x \in (xN)^2 \subseteq x^2N$ implies immediately that $x = x(x^k y) = x^{k-1}(xz)y$ $= x^{k-1}(x^{k+1}yx)zy = x^{k-1}yz(x^{k+1}yx) = x^{k-1}y(zx) = (x^k y)z$, that means $x^k y \in C$.

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